Performance Analysis of Discrete-time Queuing System through Arriving Decision Method

ISSN (e) 2520-7393
ISSN (p) 2521-5027
www.estirj.com

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Abstract: Queueing system plays an imperative role in the field of engineering discipline. It has huge variety of applications that are based on communication system. In communication system switching queue experience a characteristic that is observed sometimes when a message or data is being processed by server. At this scenario the next data which is more important than the other data will be served first. Due to this factor a queue is decompose into two different queues with high priority and low priority queues. Negative customers used for load balancing in the system to avoid traffic load. Using generating function approach we will analysis the system rigorously, and analyze the performance measures for number of customers present in the system will be achieved. The busy period (customer delay) and function for sojourn time of customer will also be obtained.

Keywords: *Queuing system, Markov Chain, Negative customers, Priority system, Busy Period*

1. Introduction

Discrete queuing models play an essential role in daily routine; many researchers have worked on the performance analysis of queuing models with different arriving techniques. Firstly it considers the First-Come-First-Serve (FCFS) strategy and Geo/Geo/1 input and output in the queue, a closed-form expression of the generating functions and the stationary distributions is obtained [1]. When the number of arrivals increases, service delays arise, and the performance of such delay systems varies greatly as well as customers with low data rates receive service from the server, causing system delays though it is obvious when the server is in use, it is possible that it will break down at any time [2] [3] [4].

In this system, if the server is unavailable when a client arrives, the customer either enters the orbit to retry for the same service when a specific amount of time has passed, or it exits the system without being served. After the first essential service is performed, the customer can choose whether or not to use the second optional service [5] . The batch Markovian arrival process does not allow the customers to enter the queue once the service has been started this totally affects the efficiency of the system[6]. In a discrete-time queue system with multiple working vacations by constructing a discrete-time Markov Chain, the number of customers in the system with varying in terms of priorities[7] .The investigation of scheme scheduling totally supports single and multiple-user connection depends on discrete queuing system models [8].

There has been a growing interest in queuing systems and networks with negative arrivals, along with their applications, throughout the last decade. A Markov flow of negative customers enters the system, the first negative client kills the last positive customer in line[9] . A discrete-time single-server retrial queue with geometrical arrivals of both positive and negative customers is studied in this study. If the server is found to be idle or busy, the arrival of a negative customer will cause the server to crash and terminate any positive customers currently in service [10].

2. Methodology

2.1 Queuing System

Queuing systems are comprised of one or more servers that provide services to customers in the manner in which they arrive. Almost everyone has experienced the uncomfortable time of having to wait in line for an extended period of time during their everyday activities. It is logical to believe that service should be provided to the individual who is first in line. This rule, however, may not always be applicable. Customers served at arbitrary may be served based on the server's priorities, so the customer with the greatest priority may receive service ahead of the one who has been standing in line for a long time.

The system consists of a single server and two queues, the input and output follow geometric arrivals, a typical queuing system have a single queue and single server, in which all the clients wait in a single queue, but in this system, a queue is decomposed into two separate queues the first queue will have ordinary customers and the second queue contains negative customers. Ordinary customers have high priority while negative customers have low priority in the service process.

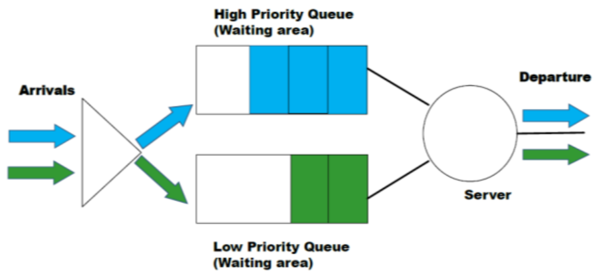

Figure.1. An illustration of a single queue split into two queues

There is four queue service discipline that a queueing system follows according to the application, here in this system the ordinary customers will stick to the First-Come-First-Serve (FCFS) technique to ensure that all the necessary information should be served first.

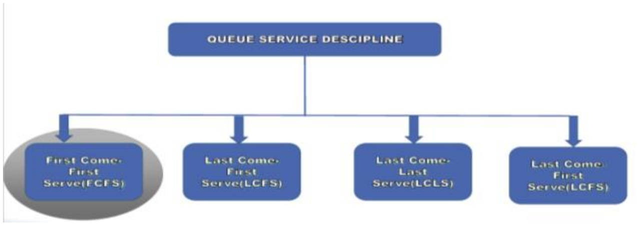

Figure.2. Service technique for ordinary customers

The Queuing system model has two arrivals a_1 and a_2, and single output β as shown in Figure 3. There are two queues Q_1 and Q_2 involved in the system, Q_1 has four customers (1 to 4) while Q_2 also had four customers (5 to 8). Q_1 Will follow FCFS strategy for service and Q_2 will be randomly served. This system includes a single server.

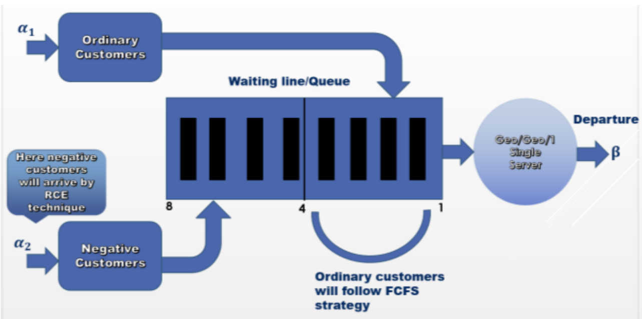

Figure.3. Queuing system model of the an entire system

In various telecommunication and computer networks, the deployment of negative customers is employed as a control mechanism. This paper investigates a discrete-time single-server queue with both positive (ordinary) and negative customers arriving geometrically. There are usually two methods for killing ordinary customers in the queue. (a)Removal of the customer at the head (RCH): It erases the ordinary customer at the service. (b)Removal of the customer at the end (RCE): It kills the most recent customer, irrespective of whether someone is in the service or waiting in a queue.

In queuing network modeling, negative arrivals have been understood as inhibitors and synchronization signals. Negative arrivals, on the surface, appear to make the system less congested than if they were not available, so their presence provides a technique for controlling excessive system congestion.

2.2 Markov Chain

The Markov chain is a random probability distribution named after Andrey Markov, a Russian mathematician. The transitions from one state to another are represented by the chain. It truly defines a series of possible events, the probability of which is determined by the state obtained in the prior event. The state space, which contains all possible future states, is a particularly useful tool in this regard. The transition or jumping occurs because, regardless of how the process arrived to its current state, the possible future states are fixed.

It also evaluates and figures out how to manage a "queue" of clients in various conditions. Internal processes, service rates, server count, queueing capacity, customer count, and queueing discipline are all features. This allows for a more accurate description and examination of the system's internal behavior.

2.2.1 Markov chain of ordinary customers

Queuing model of the Markov chain for ordinary customers is shown below in figure 4, which indicates at first there is no customer present in the queue and there are 5 states from (0 to 4). Primarily a_1 represents the ordinary customers' arrival while a_1' represents do not come, as there are many chances that they may arrive come or not. Similarly, β represents the customer got its service, and β' the customers did not get its service. First of all, at 0 state $a_1' + a_1\beta$ shows the queue is empty and the server is idle so the first customer entered in the queue. When arrival comes it always jumps to the next state (state 1) for service with $a_1\beta'$ When a_1 got his service it goes back to its same state(state 0) with $a_1'\beta$. After that, on state 1-second customers enters the queue so the further same pattern will be followed till state 4, because queue size Q_L is 4 for ordinary customers.

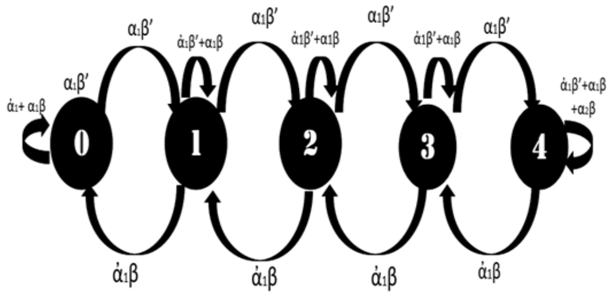

Figure.4. Markov chain of ordinary customers

2.2.2 Markov chain of negative customers

Queuing model of the Markov chain for negative customers is shown below in figure 5, which shows after 4 states all other states from (5 to 8) states are reserved for negative customers. Here a_2 represents the arrival of negative customers in the queue.

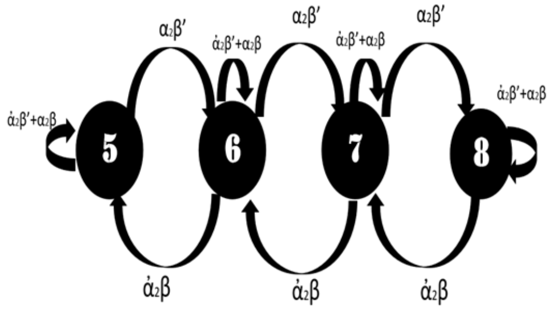

Figure.5. Markov chain of negative customers

2.2.3 Markov chain of the entire system

Queuing model of the Markov chain for all customers present in the queue is shown in figure 6, It shows all the states in the queue where Q_L=8, First four states are taken by ordinary customers while the last four states are for Negative customers. This entire chain describes the graphical representation of both the queues in one chain by the Quasi-Birth death process.

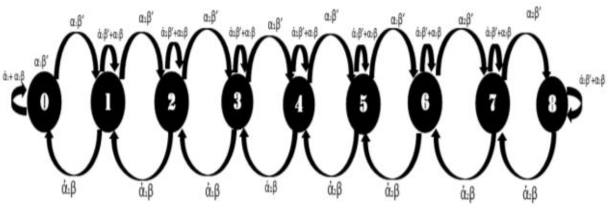

Figure.6. Markov chain of the entire system

3. Busy Period

The busy period (B.P) in a single-server queuing system is the time period during which the server is continuously occupied without a pause. It begins when a new arrival is instantly put into operation and ends when the server falls idle for the first time after that. The simple version of the busy period is the total time the system remains busy. The analytical equation is shown below.

$$\mathrm{BP} = \frac{[(1-ap_1)]^m}{1-at_1}[1 - a(p_2 + p_3) + a(p_2 + p_3)h(x)] + x\frac{1-[(1-ap_1)]^{m-1}}{1-(1-ap_1)x}[ap_2h^2(x) + ap_3h(x) + ap_4] \quad (1)$$

Where α is the probability that an arrival occurs, p_1 is probability finds the server idle, customer commences his service, p_2 represents customer finds server busy, p_3 is customer commences his service, p_4 is the probability that becomes a negative customer and $\mathrm{h}(x)$ is a probability that busy period last exactly in each slot.

4. Mean Sojourn Time

The mean sojourn time (or mean waiting time) for an object or person in a system is determined as the length of time an object or person is predicted to spend in the system before exiting permanently.

4.1 Sojourn time of customer in the Queue

The amount of time a client/customer is supposed to spend in a queue before exiting it. This time totally affects all other parameters like delay in the system, occupation of the queue for a long time, and most chance of traffic load in the queue. Following is generating function for determining the sojourn time of customers in the queue only. It is denoted by w.

$$w = \frac{[1-(ab+ap_1)x]s[([(ab+ap_1)x]ab+ap_1)x]+a(p_3+p_4)[(ab+ap_1)x-S[(ab+ap_1)x]]}{(ab+ap_1)[1-(ab+ap_1)x]-ap_2h(x)[(ab+ap_1)x-S[(ab+ap_1)x]]} - \frac{[1-(ab+ap_1)x]S[(ab+ap_1)x]+a(p_3+p_4)x-S[(ab+ap_1)x]]}{(ab+ap_1)[1-(ab+ap_1)x]-ap_2[(ab+ap_1)x-S]} \quad (2)$$

Where α is the probability that an arrival occurs, p_1 is probability finds the server idle, customer commences his service, p_2 represents customer finds server busy, p_3 is customer commences his service, p_4 is the probability that becomes a negative customer, s is generating function, $\mathrm{h}(x)$ is a probability that busy period last exactly in each slot and ab is arrival does not come.

4.2 Sojourn time of customer in the server

The distribution of a customer's spending time on the server. This time it totally depends upon the capabilities of the server and the type of server. The system with a single server doesn’t enhance the confusion for clients in the queues. It is denoted by g.

$$g = \frac{[1-(ab+ap_1)x]s[(ab+ap_1)x]+a(p_3+p_4)[(ab+ap_1)x-S[(ab+ap_1)x]]}{(ab+ap_1)[1-(ab+ap_1)x]-ap_2h(x)[(ab+ap_1)x-S[(ab+ap_1)x]]} \quad (3)$$

4.3 Sojourn time of customer in the system

The amount of time a customer spends in the system from the start of their service to the time they leave. It is the combination of the sojourn time of the customer in the queue and in the server. It represents as:

$$b = w + g \tag{4}$$

Following is generating function for determining the sojourn time of the customer in the system.

$$b = \frac{[1-(ab+ap_1)x]S[(ab+ap_1)x]+a(p_3+p_4)x-S[(ab+ap_1)x]]}{(ab+ap_1)[1-(ab+ap_1)x]-ap_2[(ab+ap_1)x-S[(ab+ap_1)x]]} \tag{5}$$

5. Stationary distribution

A stationary Markov chain distribution is a probability distribution that does not vary over time in the Markov chain. It is often expressed as a row vector π whose elements are probabilities aggregating to 1. The stationary distribution of the Markov chain has the following two generating functions, one is of Queue size and the other is system size.

5.1 Queue Size

The Probability generating function of the queue is represented as:

$$E[N] = \frac{1}{[p_2\, S(ab+ap_1)+(p_3+p_4)(ab+ap_1)]D(1)} * \{[p_2(S(ab+ap_1)+ap_1S(ab+ap_1)+ap_1(p_3+p_4)-(ab+ap_1)(p_1+p_2)]D(1)-[p_1s(ab+ap_1)+(p_3+p_4)(ab+ap_1)]D(1)\} \tag{6}$$

Where $D(1)$ is the stability factor

5.2 System Size

The Probability generating function of the system is represented as shown in equation 6:

$$E[L] = E[N] + 1 - \pi_0 \tag{7}$$

Where

$$\pi_0 = \frac{D(1)}{p_1 s(ab+ap_1)+(p_3+p_4)(ab+ap_1)}$$

Where α is the probability that an arrival occurs, p_1 is probability finds the server idle, customer commences his service, p_2 represents customer finds server busy, p_3 is customer commences his service, p_4 is the probability that becomes a negative customer, s is generating function, and ab is arrival does not come.

6. Results and Discussion

The results are carried out on MATLAB software, the busy period is calculated in such a way as shown in figure 7, when the first customer begins their service, the system will take the longest time and will be reserved for a long period, however, the second customer will take less time than last, due to that system time will decrease as each customer arrives.

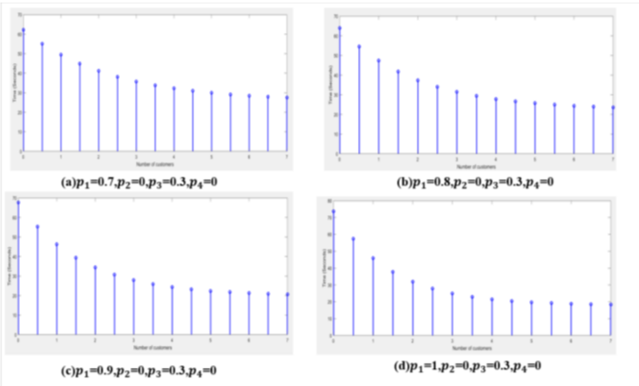

Figure.7. Busy period

The sojourn time of customers in the server is calculated by using MATLAB software shown in Figure 8, as the number of customers' increases, the server becomes busy for a long time so the sojourn time will rise with the number of customers. Similarly, in Figure 9, time in queue is directly proportional to the number of clients as one unit increases other also rises due to the queue being reserved for a long time.

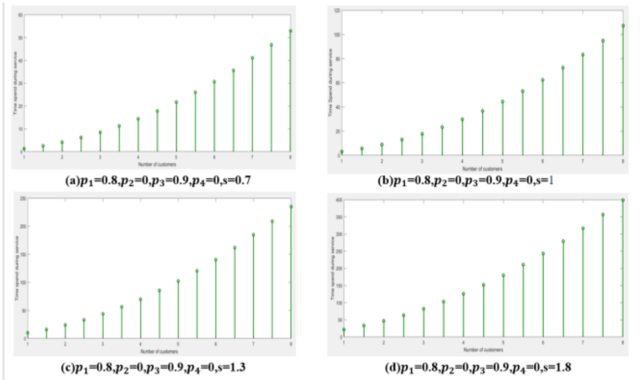

Figure.8. Sojourn time in server

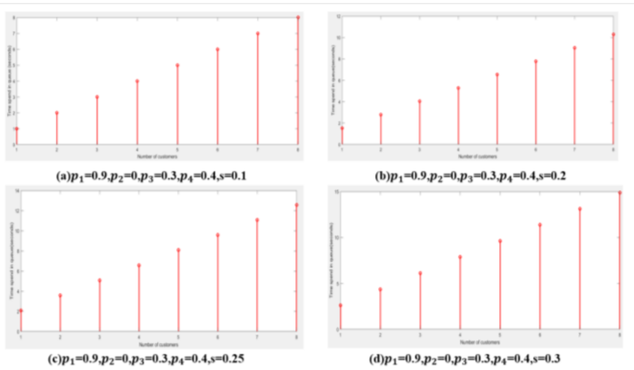

Figure.9. Sojourn time in queue

The arrival rate versus the overall number of clients in the system is plotted shown in Figure 10. The result was carried out on a system with a limited number of customers. As we can see, as the number of clients grows, so does the service rate.

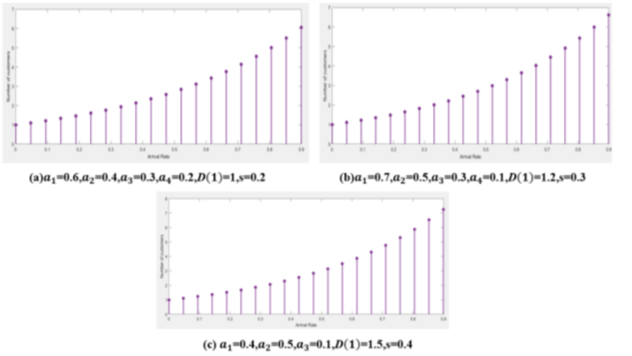

Figure.10.Service rate of the system

7. Conclusion

This research considers a discrete system queueing model with the geometric distribution. We used geometric distribution and a priority-based method to illustrate the queue's arrival process. Negative customers are important in discrete queuing modals because they provide the system with synchronization signals.

Congestion/traffic load is minimized by putting negative customers into the system, and when a customer is missing in line, the lost client is identified using the RCE technique. The system's reserve time will be known, and sojourn time will be observed by generating functions after the busy period is computed with varied rates of probabilities.

Finally, in a discrete-time system, the service rate is improved with arrival modals, demonstrating that as the number of consumers in the system grows, so does the efficiency.

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