Service and Flow Processes Analysis of Congested System using Double Service Rate

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Abstract: In this paper service and flow process analysis of queuing system is conducted, aiming to minimize the congestion in the queuing system by applying double service rate to provide service at maximum possible level. Arrival process is taken as Poisson distribution and service process is exponentially distributed. System has only one server with finite queue size with two thresholds Q_1 and Q_2. Queuing system has two types of service which associates with the queue thresholds, system has simple service rate when queue size is $\leq Q_1$, when queue size is $>Q_1$ the system will switch to the double service rate in order to minimize the congestion in the system. A queuing system is modeled with only one server. Markov process chain and flow process of the system is generated along with its transition matrix and starting states equations. Congestion of the system can be easily handled by increasing the service rate when the number of customers reach a particular threshold.

Keywords: *Queuing System, Flow process, Markov process, Service Rate, Arrival Rate*

1. Introduction

Queuing theory is mathematical probabilistic modeling tool to understand and figure out a way to handle a "queue" of clients in different scenarios [4,5,8]. A queue is formed whenever service requests or customers arrive at a service facility and are forced to wait while the server is busy working on other requests. The basic operations in a queuing system are input or arrival process, output or service process, number of servers or service channels, queue discipline, system Capacity and customer behavior.

Some of the important applications of queuing modeled system are in communication systems, stocking systems production systems, information processing systems and transportation.

In this paper the idea of double service rate is explored which can be applied to the different available queuing systems to improve the performance of the system. The idea is applied on sample system of M/M/1/C/FCFS where customers arrival process is according to a Poisson process at rate λ and the service customers receive is exponentially distributed with a mean service time of μ from a single server with limited queue size C and served on first come first served basis.

Congestion occurs when the system or server has more customers than its ability to handle which results in balking reneging and delays [3,10]. Double service rate has two types of service probabilistic thresholds for managing the load which depends on internal thresholds of the system.
The service and flow process analysis of the system provide the detailed information about the customer in a system that which path and how much time it will take to flow in the system [4,5,11].Service and flow process analysis determines the congestion occurrence in the queuing system in order to switch the service rate to the second probabilistic service rate which is double service rate. This results in less congestion by handling traffic efficiently, increases system efficiency and ability to provide service at maximum possible level.

2. Queuing System

A queuing system basically consists of one or more servers that provides services to arriving customers in the system. Customers which arrives to find all the servers in the system busy generally joins one or more queues which are placed in front of the servers, hence the name queuing systems [2,3,5]. Queuing system are characterized by inter arrival rate, service rate, number of servers, queuing capacity, number of customers and queuing discipline. single node queuing systems are shorthand by Kendall's notation as A/B/C/k/m/z

A denotes the distribution of inter-arrival times which can be Exponential, Poisson, Erlang, Deterministic, General.
B represents the distribution of service time which can be Exponential, Poisson, Erlang, Deterministic, General.
C denotes the number of servers.
k denotes the queuing capacity finite or infinite.
m denotes the population of customers finite or infinite.
z denotes the queuing discipline FIFO, LIFO, SIRO.

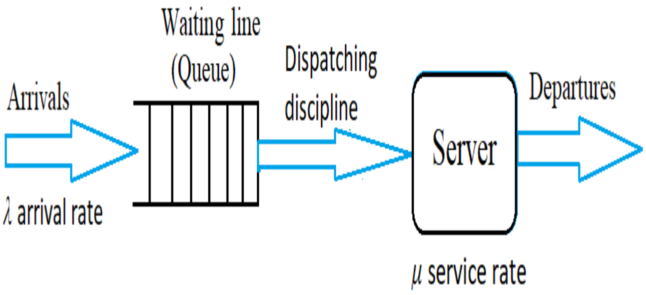

Figure.1. Basic Queuing System

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3. Methodology

3.1 System model

In this paper, service and flow process analysis of a queuing system is presented. Double service rate is applied on queuing system of M/M/1/C/FCFS where customers in the system arrives at rate λ according to a Poisson process. Receives service with the service rate of μ that is exponentially distributed from a single server with limited queue size C and served on first come first served basis. Double service rate has two types of service probability thresholds for managing the load which depends on internal thresholds of the system.

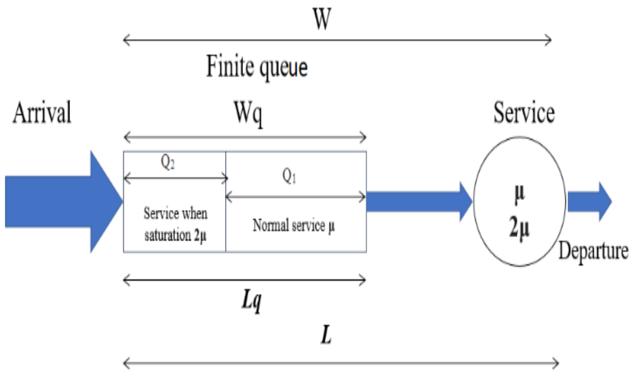

Figure.2. Queuing System model

λ is Poisson arrival rate of customers/time unit where as μ is mean service rate of customers/time unit exponentially distributed, L defines the average number of customers in the system (server + queue), L_q shows the average number of customers in the queue. W is the average delay in the system (server + queue), W_q represents the average delay in queue. Queue has two thresholds when customers in the queue Care $\leq Q_1$threshold the queuing system services at normal service rate μ. When customers in the queue C are $>Q_1$ threshold then system will switch to second probability service with the service rate of 2μ.

3.2Process Markov Chain

The stochastic process X(t) is said to be a Markov process if it satisfies the Markovian property which is the memoryless property

$$P\{X(t_n+1) = x_n+1 \mid X(t_n) = x_n \ldots\ldots X(t_1) = x_1\} = P\{X(t_n+1) = x_n+1 \mid X(t_n) = x_n\}$$

This is known as the memoryless property because the state of the system at future time t_{n+1} is only decided by the system state at the present time t_n and it does not depend on the statet_1, t_2,t_3…..,t_{n-1} which are earlier time instants.

The chain consists of 6 states S1, S2, S3, S4, S5, and S6 there are 6 number of customers out of which 5 customers are in queue and 1 customer in server. The arrival rate of customers is λ and service rate of customers are μ and 2μ.

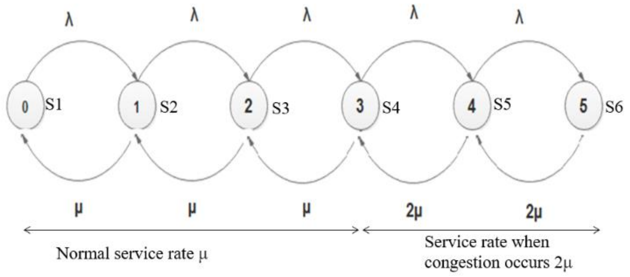

Figure.3. Process Markov chain

At state S1 when customer arrive in the system it finds the queue empty and server idle it goes direct to the server. At state S2 another customer arrives in the system it finds the server busy serving the first customer the second customer waits in the queue at state S2total 2 customers in the system1 of which is in the queue and 1 in the server. Similarly, at state S3 total number of customers are 3 in the system out of which 2 customers are waiting in queue and 1in the server. At state S4 total number of customers are 4 in system out of which 3 customers are in queue and 1 in the server. At state S5 total number of customers are 5 out of which 4 customers are in queue and 1 in the server. At state S6 total number of customers are 6 out of which 5 customers are in queue and 1 in the server. System will have normal service rate μ for state S1 to state S4 as system is not congested the system handles customers with normal service rate. If system is at state S4 and another customer arrive system becomes congested the system will start serving at full capacity with double service rate 2μ for the state S5 and S6.

State transition matrix of the above system is calculated as

$$P = \begin{bmatrix} 0 & \lambda & 0 & 0 & 0 & 0 \\ \mu & 0 & \lambda & 0 & 0 & 0 \\ 0 & \mu & 0 & \lambda & 0 & 0 \\ 0 & 0 & \mu & 0 & \lambda & 0 \\ 0 & 0 & 0 & 2\mu & 0 & \lambda \\ 0 & 0 & 0 & 0 & 2\mu & 0 \end{bmatrix}$$

3.3Flow and service process

The flow process of a customer is determined by the insertion of test data packet into the system. Flow process provides the graphical illustration of flow of each customer in the system. The flow process shows all feasible flow paths of customers as well as the entering states where customers are allowed to starts the flow into the system and leaves the system when it receives the service. It analyzes the entire flow process path of the test data packet, when the packet enters into system receiving the service and departs from the system. Flow process of the system gives insight of system state behavior and calculate the chances of occurrences of customer at any state except the blocking state. Flow process Markov chain is constructed from system states and service process Markov chains. Transition diagram for the flow process is based on three different states which are (1) flow process starting states at these states data packet arrives into the system (2) indirect states where the flow process does not start is known as indirect states(3) absorbing states where the flow process of the system ends is known as absorbing states.

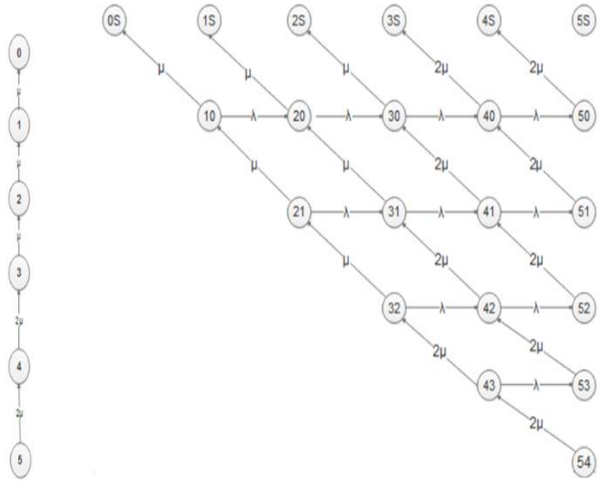

Figure.4. Flow Process and Service Process

States10, 21, 32 and 43 are the starting states of the system where the customer enters into the system where as state 54is the blocking state where system cannot take more customers and arriving customers will be dropped. Equations of starting states probabilities of the system for the arrival of test data packet to start its flow process are

$$\sigma_{10} = \frac{\pi_{10}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}} * \frac{1}{1-\frac{\pi_{10}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}}}$$

$$\sigma_{21} = \frac{\pi_{21}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}} * \frac{1}{1-\frac{\pi_{21}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}}}$$

$$\sigma_{32} = \frac{\pi_{32}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}} * \frac{1}{1-\frac{\pi_{32}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}}}$$

$$\sigma_{43} = \frac{\pi_{43}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}} * \frac{1}{1-\frac{\pi_{43}}{\pi_{10}+\pi_{21}+\pi_{32}+\pi_{43}}}$$

4. Results and Discussion

Programs are written in MATLAB for analytical and simulation purpose for calculation of probability density function and flow time. Flow process calculate the chances of occurrences of customer at any state except the blocking state. We have taken four different scenarios in which service rate changes according to the number of arriving customers. Probability density function of starting states and total probability density function with different arrival rates are calculated.

Scenario 1: When queue threshold ≤ Q1 arrival λ=0.4 so service rateμ=0.3 decreased arrival so normal service rate figure 5 illustrate the scenario 1

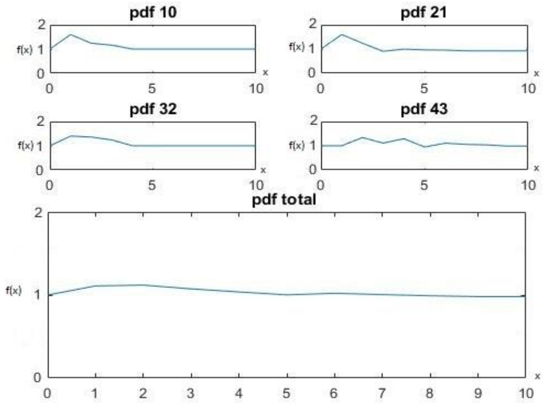

Figure.5. Scenario1 arrival λ=0.4 so service rateμ=0.3

Scenario 2:When queue threshold ≤ Q1 asarrival λ=0.5 so service rate μ=0.5 decreased arrival so normal service rate Figure 6 illustrate the scenario 2

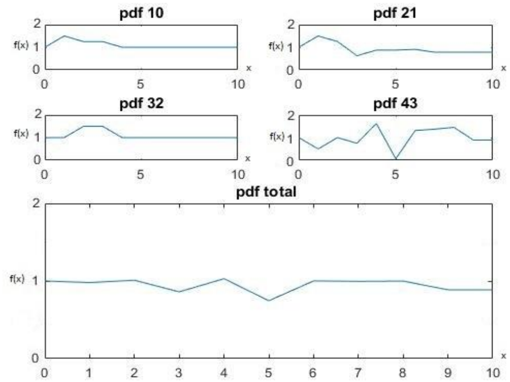

Figure.6. Scenario 2 arrival λ=0.5so service rateμ=0.5

Scenario 3: When queue threshold >Q1 as arrival λ=0.6 so double service rate μ=1.2 arrival increases congestion occur so double service rate Figure 7 illustrate the scenario 3

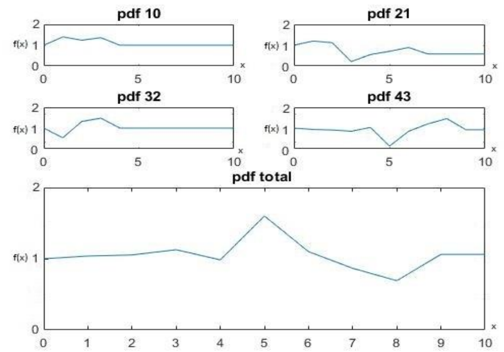

Figure.7. Scenario 3 arrival λ=0.6 so double service rate μ=1.2

Scenario 4: When queue threshold >Q1 as arrival λ=0.8 so double service rate μ=1.6 arrival increases congestion occur so double service rate Figure 8 illustrate the scenario 4

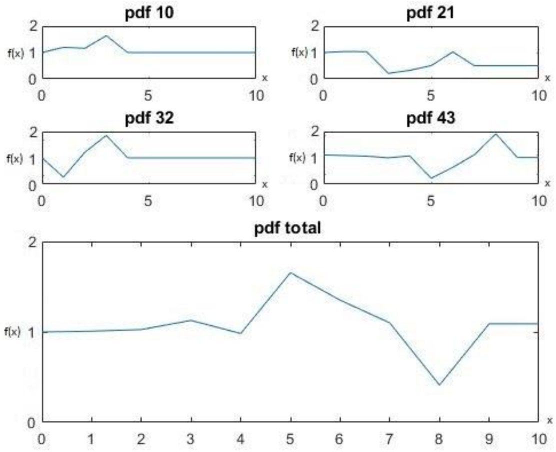

Figure.8. Scenario 4 arrival λ=0.8 so double service rate μ=1.6

Probability density function of entering states 10,21,32 and 43 are shown in the upper parts of figure 5,6,7 and 8 the lower parts shows the total probability density function. Obtained analytical and simulation results shows that when congestion in the system occurs the modeled system provides double service rate to minimized the congestion in the system.

5. Conclusion

In this paper service and flow process analysis of queuing system is discussed. Process Markov chain was constructed and its transition matrix calculated. Markov flow process was constructed which illustrated the starting states, indirect states and absorbing states. Starting states probabilities of the system were calculated. Probability density function of starting states and total probability density function was illustrated with the simulation results. From analytical and simulation result it is eminent that double service rate minimized the congestion in the system. Double service rate resulted in less congestion by handling traffic efficiently and provided service at maximum possible level. Congestion of the system can be easily handled by increasing the service rate when the number of customers reached at a particular threshold.

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