

# **SumRate Performance of Precoding Techniques in Multiuser MIMO Systems**

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Abstract: Wireless Communication has attained rapid development in every field of communication science and technology. It has changed and revolutionized many applications and brought innovative changes in respective applications. There have been always upcoming demands for high data rates because of user demand and bandwidth hungry wireless devices. Multiple Input and Multiple Output (MIMO) addresses the issue of high data rates and achieve unique diversity by using same bandwidth and power, allowing system to transmit more data bits. Coding schemes are used to handle multiple users in MIMO systems. Dirty Paper Coding is a nonlinear technique that achieves MIMO Broadcast Channel capacity but the complexity of this scheme makes it difficult to implement. In contrast, there are some linear pre-coding techniques such as channel inversion and zero-forcing pre-coding which are easier to implement and perform well but these techniques are not capacity achieving. This leads to investigation of other nonlinear pre-coding methods like Tomlinson-Harashima Precoding and Vector Perturbation Pre-coding (VPP) which can achieve performance close to DPC. This paper analyzes the performance of DPC, Zero-forcing Beamforming and Vector Perturbation Pre-coding in terms of Sum rate and compares the effect of increase in number of antennas on each technique.

Keywords:MIMO systems, Broadcast channel, Precoding, Dirty Paper Coding, Vector Perturbation Precoding

# **1. Introduction**

Wireless communications is a rapidly growing field of the communications industry. Cellular systems have experienced exponential growth over the last couple of decades. Cellular phones have become an important business tool and part of everyday life and are rapidly replacing wire line systems in many developing countries. Wireless local area networks have replaced wired networks in many homes, businesses, and campuses. The explosive growth of wireless systems has resulted in the rapid increase in number of wireless devices such as smart phones, laptop and palmtop computers. Ever growing demands for higher throughput from a wireless link indicate that in order to provide the performance which is necessary for these emerging applications, there are many technical challenges concerned with the design of robust wireless networks.

This is a well-known fact that multiple-input multipleoutput (MIMO) systems can provide large bit rates as compared to the single antenna systems. For single-user MIMO system with independent and identically distributed (i.i.d.) Rayleigh fading channels, the capacity scales linearly with the minimum of the number of antennas at transmitter or receiver. Multiple antenna technology has attracted attention in wireless communications because it offers significant improvement in data rate and link range with the same bandwidth and transmit power as for the single antenna system. The high data rate offered by MIMO systems is due to the use of space (or antenna) diversity at both the transmitter and the receiver.

# 2. Related Work

There has been a lot of research in single-to-multipoint communication, for example when a base station communicates with several mobile devices in the same cell, which shows that by exploiting the spatial diversity of receivers, desired multiplexing gain can be achieved despite the fact that each receiver may have one antenna only. This technique by which users are separated in space is commonly termed as spatial division multiple access (SDMA). In a MIMO system with N<sub>T</sub> number of antennas when a user sends the data, the data is divided into  $N_T$ separate sub-streams. These sub streams of data are then encoded into channel symbols. Generally, all transmitters have the same data rate but different data rates can be provided to each of the sub-stream using adaptive modulation. The signals are received by N<sub>R</sub> receive antennas. This transmission scheme provides a linear increase in spectral efficiency. Traditional systems which do not utilize diversity or utilize only receive diversity provide a logarithmic increase single receiver in same frequency band, the channel is termed as multiple access channel (MAC). Conversely, the downlink setting where a single transmitter communicates with several receivers in the same frequency band sharing the same medium is termed as broadcast channel (BC) [3]. This paper focuses on the MIMO BC settings. In order to serve multiple users simultaneously in a multiuser MIMO system, there is a need of an efficient encoding scheme. The capacity region

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of MIMO BC is achieved by using dirt-paper-coding (DPC) at transmitter and SIC receiver at each user [4]. As the complexity of DPC makes it difficult to implement, some linear precoding techniques such as channel inversion and zero-forcing precoding were implemented. These techniques performed well but could not achieve capacity near to DPC. Therefore, nonlinear precoding methods like Tomlinson-Harashima Precoding [5] and more promising Vector Perturbation Precoding (VPP) were investigated.

## 3. Methodology

#### 3.1 System model

A single-cell MIMO Broadcast Channel with a single base station is considered. The base station is assumed to be equipped with M transmit antennas and communicates with K users. Each user terminal is assumed to have single receive antenna i.e.  $N_k=1$ . The channel gain from a transmit antenna to a user is modeled as a zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variable. It is assumed



Fig. 2. System model of Multiuser MIMO Broadcast channel

that all the users are at the same distance from the base station and encounter independent fading. The signal received by the kth user may be written as:

$$y_k = H_k x + n_k k = 1, \dots, K$$
 (1)

where  $\mathbf{x} \in \mathbb{C}^{M \times 1}$  is the transmitted symbol,  $\mathbf{H}_k \in \mathbb{C}^{N_k \times M}$  is the channel gain matrix to the  $k^{\text{th}}$  user,  $\mathbf{n}_k \in \mathbb{C}^{N_k \times 1}$  is the additive white Gaussian noise (AWGN) at the kth user and  $y_k$  is the signal received by user k. Further, it is assumed that the perfect Channel state information (CSI) is present at the base station.

#### 3.3 Single User MIMO Systems

While most wireless systems today support multiple users, single-user results are still of much interest for the accurate and deep understanding they provide and their application to channelized systems where users are allocated orthogonal resources (time, frequency bands, etc.). MIMO channel capacity can also be derived easily for single users than for multiple users. There are many cases for which multiuser problems are still unsolved whereas the corresponding single user problems have already been solved and the results are already known. Consider a transmitter with M transmit antennas, and a receiver with N receive antennas. The channel can be represented by the NxM matrix H. The Nx1 received signal y is equal to

$$y = Hx + n$$

(2)

Where x is the M x 1 transmitted vector and *n* is Nx1 additive white circularly symmetric complex Gaussian noise vector, normalized so that its covariance matrix is the identity matrix. The normalization of any non-singular noise covariance matrix  $K_n$ to fit the above model is as straight forward as multiplying the received vector y with  $Kn^{-1/2}$  to yield the effective channel  $Kn^{-1/2}H$  and a white noise vector.



Fig. 1. General model of a MIMO System with  $N_t$  transmit antennas and  $N_r$  receive antennas

#### 3.2 Multi-User MIMO Systems

In MIMO systems, multiple-users share the same channel without any increase in bandwidth, as required in time, frequency or code-division multiple access schemes. Multiple transmit antennas are used to generate enough spatial degrees of freedom to separate the signals so they are received with little or no interference at each user. MU-MIMO has two types: one is the uplink channel and the other is downlink channel. The downlink channel is known as the MIMO Broadcast Channel where different information streams are sent to the users by the base station. On the uplink, different information from the users is received by the base station. Since multiple users access the base station on the uplink channel, therefore it is also referred to as Multiple Access Channel. In the uplink, because the receive antennas can cooperate, similar detection techniques for point-to-point MIMO spatial multiplexing systems can be applied [16].



Figure. 3. Multi-User Mimo System [9]

Of the two channels, BC and MAC, broadcast channel is more challenging within MU-MIMO because the users cannot cooperate and the multi-user interference is to be considered at the transmitter side. This requires transmit processing which is typically in the form of pre-coding and SDMA, Space Division Multiple Access based downlink user scheduling. Channel State Information at the Transmitter, CSIT is necessary for this purpose. This provides significant improvements in throughput as compared to that of ordinary point to point MIMO systems. Also, the performance is more enhanced when the number of transmit antennas is greater than the number of antennas at each receiver. On the downlink, the MIMO sum capacity scales with the minimum of the number of base station antennas and the sum of antennas at each user terminal. This means that capacity gains can be achieved in MU-MIMO with a multiple antenna base station and multiple single antenna mobile users.

## 4. Precoding Techniques In Multi-User Mimo Systems

Precoding means that multiple data streams are emitted from the transmit antennas with independent and appropriate weightings in order to maximize the link throughput at the receiver output. This benefit of precoding can be realized without requiring channel state information at the transmitter. Most of the classic results related to precoding assume narrowband, slowly fading channels which means that for a certain period of time the channel can be described by a single channel matrix which does not change faster. In multi-user MIMO, since the base station communicates with multiple users, the data streams are intended for different users (known as SDMA) and the goal is to maximize some measure of the total throughput (e.g., the sum performance). For SDMA systems, precoding algorithms are further divided into linear and nonlinear precoding types. The capacity is achieved by nonlinear algorithms but these algorithms are much complex. In contrast, linear precoding approach usually achieves reasonable performance with much lower complexity. Nonlinear precoding is based on the idea that if the interference is already known at the transmitter, the transmitter can pre-cancel this interference by applying an optimal precoding scheme on the transmit signal without the penalty of radio resources. This concept was introduced as Dirty Paper Coding. Reduction in complexity while simultaneously keeping the error rate low will help reduce power consumption, and hence increase the operation time for mobile receivers A reduction in receiver complexity can achieved by moving some signal processing be functionality from the receiver to the transmitter. This transfer of signal processing functionality from the receiver to the transmitter is referred to as transmitter based preprocessing. This results in complicated transmitters. However, in many systems a complicated transmitter can be allowed. A typical example is the downlink of a cellular communication system or a wireless access point, deployed in an indoor wireless local area network (LAN). The transmitter in these cases can accommodate enhanced complexity because virtually an infinite amount of energy is available for signal processing. Cellular phones in case of the cellular downlink and mobile handheld devices, wireless access cards used in lap- tops in case of wireless LANs, are preferred to have low power consumption and compact size. Transmitter based pre-processing techniques are most suitable for such applications. Precoding is one of these techniques.

#### 4.1 Zero Forcing Beamforming

Consider a system model of a linear precoder assuming that  $N_k = 1$ . The received signal (now a scalar) can be written as

$$\mathbf{y}_{\mathbf{k}} = \mathbf{h}_{\mathbf{k}}\mathbf{x} + \mathbf{n}_{\mathbf{k}} \tag{3}$$

$$= \sqrt{P_k} \boldsymbol{h}_k \boldsymbol{f}_k a_k + \sum_{j=1}^n \sqrt{P_j} \boldsymbol{h}_k \boldsymbol{f}_j a_j + n_k$$

wherea<sub>k</sub>,  $P_k$ ,  $f_k$  and  $n_k$  are the data symbol, allocated power, precoding vector and noise for user k respectively. The maximum achievable sum rate for the this linear precoding vector is given by

$$R_{Lin} = \max_{f_{k}, p_{k}} \sum_{k=1}^{K} \log \left( \frac{1 + \sum_{j=1}^{K} P_{j} |h_{k}f_{j}|^{2}}{1 + \sum_{j=1}^{K} p_{j} |h_{k}f_{j}|^{2}} \right)$$
(4)

Zero-forcing Beamforming (ZF-BF) is a spatial signal processing in which the data is transmitted to desired users in such a way as to maximize the gain in the direction of intended users and nulling out the gain in the directions of undesired users. Similarly, for uplink, ZF-BF receives from the desired users and nulls out the directions from the interfering users. In ZFBF, the precoding vector  $f_k$  is chosen in such a way that results in zero interference at other users i.e. hkfj=0 for  $j\neq k$ . For the case when  $K > N_T$  a user selection algorithm is used to select  $K \leq N_T$  users. Let  $S = \{1, \ldots, K\}$  where  $|S| \leq N_T$  be the set of selected users. In this case the ZFBF matrix will be given by

$$\mathbf{F}(\mathbf{S}) = \mathbf{H}^+ = \mathbf{H}(\mathbf{S})^{\dagger} (\mathbf{H}(\mathbf{S})\mathbf{H}(\mathbf{S})^{\dagger})^{-1} \qquad (5)$$

The achievable rate for ZFBF is given by [23]

$$R_{ZFBF}(S) = \max_{\sum_{k \in S} \gamma_k P_k < P} \sum_{k \in S} \log(1 + P_k)$$
(6)

where

$$\gamma_k = \frac{1}{\|\mathbf{f}_k\|^2} = \frac{1}{\left[(\mathbf{H}(S)\mathbf{H}(S)^{\dagger})^{-1}\right]_{k,k}}$$
(7)

is the effective channel gain for the  $k^{\text{th}}$  user. In order to maximize the achievable sum rate the optimal power allocation  $P_k$  can be found by waterfilling as

$$P_k = (\mu \gamma_k - 1)^+ \tag{8}$$

where  $x^+$  is max(0, x) and  $\mu$  is the water level calculated by solving  $\sum_{k \in S} \left( \mu - \frac{1}{\gamma_k} \right) = PP$ . The achievable sum rate of ZFBF is found by considering every possible choice of user groups S [23]

$$R_{ZFBF} = \max_{S \in 1, \dots, K, |S| \le N_T} R_{ZFBF}(S)$$
(9)

#### 4.2 Dirty Paper Coding

The capacity region of MIMO BC is the set of all achievable rate regions, it can be mathematically written as [4,20]

$$\mathbf{R}(\boldsymbol{\pi}, \mathbf{Q}_{1}, \dots, \mathbf{Q}_{K}) = [R_{1}, \dots, R_{K}]'_{K}]'$$
(10)

where

$$R_{k} = \log \frac{det \left[ \mathbf{I} + \mathbf{H}_{k} \left( \sum_{i=1}^{k} \mathbf{Q}_{\pi(i)} \right) \mathbf{H}_{k}^{\dagger} \right]}{det \left[ \mathbf{I} + \mathbf{H}_{k} \left( \sum_{i=1}^{k-1} \mathbf{Q}_{\pi(i)} \right) \mathbf{H}_{k}^{\dagger} \right]}$$

where  $\pi$  is the permutation function on the user set 1, . . . ,K and  $Q_k$  is a positive semi definite transmit covariance matrix of user k satisfying the power constraint.

$$\mathsf{E}[\mathsf{x}\mathsf{x}^{\dagger}] = \mathsf{tr}\{\sum_{k=1}^{K}\mathsf{Q}_{k}\} \le \mathsf{P}$$
(11)

Then, the capacity region of the MIMO BC (achieved by DPC), is given as the convex hull of the union of all such rate vectors over all permutations and all positive semi-definite matrices[4,20].

$$C_{DPC} = \operatorname{conv}\left(\bigcup_{\pi, \operatorname{tr}\{\Sigma_{k=1}^{K} \mathbf{Q}_{k}\} \le P, \mathbf{Q}_{k} \ge 0} \mathbf{R}(\pi, \mathbf{Q}_{1}, \dots, \mathbf{Q}_{K})\right)$$
(12)

#### 4.2 Vector Perturbation Precoding

Vector perturbation (VP) precoding [6] is a promising practical transmission method which performs better than linear precoders. With VPP, the data vector to be transmitted is constrained to lie within a 2K-dimensional hypercube of side length one, the addition of a perturbation vector modifies the data vector before being passed through a channel inverting linear precoder. The perturbation vector consists of complex integers and the power required to transmit the signal is notably reduced by the addition of the perturbation vector.

The sum rate  $R_{VP}$  of an  $N_T \times K$  vector perturbation system with uniformly distributed inputs is [1]

$$R_{VP}(\mathbf{H}, \mathbf{F}) \triangleq \sum_{k=1}^{K} I(\hat{a}_{k}; a_{k} | \mathbf{H}, \mathbf{F})$$
$$= K \log \frac{P}{K} - K \log \frac{\pi e \varepsilon_{se}(\mathbf{F})}{K} + 2K\Omega\left(\frac{\varepsilon_{se}(\mathbf{F})}{2P}\right) \quad (13) \setminus$$

### 4. Results and Discussion

A single-cell MIMO Broadcast Channel with a single base station is considered. The base station is assumed to be equipped with M transmit antennas and communicates with K users. Each user terminal is assumed to have single receive antenna i.e.  $N_k=1$ . The channel gain from a transmit antenna to a user is modeled as a zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variable. It is assumed that all the users are at the same distance from the base station and encounter independent fading. The signal received by the k<sup>th</sup> user may be written as:

$$\mathbf{y}_{\mathbf{k}} = \mathbf{H}_{\mathbf{k}}\mathbf{x} + \mathbf{n}_{\mathbf{k}}k=1,...,\mathbf{K}$$
(14)

The sum rate of Zero-forcing beam forming is calculated and plotted in Fig.3 .The power is equally divided among the users such that if  $P_i$  be the power allocated to i<sup>th</sup> user then Pi=P/K. In Fig. 3 it is shown that at an SNR of 20 dB with M=K=6, sum rate is 21 bps/Hz which can be achieved by DPC even with M=K=4 as shown in Fig. 4.



Fig. 4. Sum rate performance of ZFBF as a function of SNR with Uniform power allocation, M=K=6 and M=K=10.

The base station is assumed to have four antennas i.e. M=4 transmitting to K≤4 users simultaneously. There is a total transmit power constraint of P=10 dB. In Fig.5, sum rate is plotted as a function of number of users. Since there are four transmit antennas, the base station can transmit to less than or equal to four users. The user set is chosen by performing an exhaustive search over the entire user set. Fig. 4 shows that the sum rate of optimal ZFBF is 12.5 bps/Hz which is greater than the sum rate of ZFBF with equal power distribution. It should be noted that the sum rate of optimal ZFBF is better even with small number of antennas i.e. M=K=4 whereas the sum rate of ZFBF with equal power distribution is approximately 7 bps/Hz with M=K=6.



Fig. 5. Sum rate performance of optimal ZFBF as a function of number of users K with M=4 and P=10 dB

The sum rate of DPC is calculated using (6).Optimal power is found by water filling algorithm. Sum rate is plotted as a function of SNR and is calculated for M=K=2 and M=K=4. Fig. 5. shows that with an increasing SNR, the performance of DPC is also increasing if there is an increase in the number of antennas and users. However, the performance of DPC is limited due to its computational complexity.



Fig. 6. Sum rate performance of DPC using water filling algorithm with M=K=2,4

The sum rate of VPP is calculated using (7). Sum rate is plotted as a function of SNR with M=K=2 and M=K=4. Fig. 6 shows that the sum rate is near to the sum rate of DPC shown in Fig. 5 and it is to be noted that there is a

large difference between the sum rate of VPP with M=K=2 and M=K=4 which means that the performance of VPP also increases with increasing the number of antennas and/or users.



Fig. 7. Sum rate performance of VPP with M=K=2,4

The sum rate of ZFBF and DPC using water filling algorithm is plotted in Figure 4.5 with M=K=4. The performance of ZFBF-WF is compared with the sum rate performance of DPC. It is clear from Figure 4.5 that there is a large gap between the performance of DPC and ZFBF-WF for the whole range of SNR.



Figure 8 Sum rate performance comparison of DPC and ZFBF-WF with M=K=4

Fig. 8. illustrates the sum rate performance of DPC and ZFBF waterfilling with M=K=6. It can be observed from Fig. 7. that there is a prominent increase in the sum rate of DPC as the number of antennas is increased but the performance of ZFBF is same as with M=K=4.



Fig. 9.Sum rate performance comparison of DPC and ZFBF-WF with M=K=6

Fig. 9.illustrates the sum rate performance comparison of DPC and VPP with M=K=2.Sum rate of DPC and VPP is calculated using (6) and (7) respectively. The plots are obtained by using one thousand independent channel realizations. In Fig. 8. it is clear that the sum rate of VPP is near to DPC for whole range of SNR. It should be noted that this sum rate performance is calculated for two transmit antennas and two user i.e. M=K=2.

However, a significant improvement in performance can be observed by increasing the number of antennas which is illustrated in Fig. 10.



Fig. 10 Sum rate performance comparison of DPC and VPP with M=K=2

Fig. 11.illustrates the sum rate performance comparison of DPC and VPP with M=K=4. It is clear from Fig. 10. that there is an improvement in the sum rate of DPC and VPP as the 42 number of antennas and users is increased from 2 to 4. Also, the sum rate of VPP approaches DPC as the SNR increases.



Fig. 11.Sum rate performance comparison of DPC and VPP with M=K=4

#### **5.** Conclusion

In a practical system the number of users may be in order of tens or hundreds. It is clear from (6) that the sum rate calculation of DPC involves a large number of permutations and in order to choose a code word for a particular user the base station has to perform the search over entire space of covariance matrices for the entire set of users. This is the reason for which DPC is hard to implement in practical systems. Hence it is concluded that linear precoding techniques are easy to implement but their performance is much lower than DPC. On the other hand, VPP achieves capacity that is near to DPC.

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